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NUMERICAL SIMULATION OF SUPERSONIC FREE SHEAR LAYERS

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INTRODUCTION

The objective of the current research is to study the mixing and stability characteristics of three dimensional supersonic free shear flows through a direct numerical solution of 3-D compressible flow equations. Supersonic shear flows are of interest to SCRAMJET engine designers. The successful operation of these engines requires rapid and efficient (i.e. minimum total pressure loss) mixing of supersonic airstreams and subsonic/sonic fuel streams.

The present research effort spans a four year period (February 1987-November 1990), and this progress report covers the period December 1, 1990 - May 31, 1990. It may be recalled that the following accomplishments were reported during the previous research periods:

i) During the period February 1987-November 1988 work focused on the stability and growth characteristics of 2-D supersonic shear layers. After experiments with 2-D implicit time marching algorithms of second order spatial accuracy and first order time accuracy, an explicit time marching scheme patterned after the work of Turkel and Bayliss at NASA Langley was developed. This method is fourth order accurate in space and second order accurate in time, and was used to study effects of convective Mach number on 2-D shear layer growth, and the response of 2-D shear layer to single and multi-frequency acoustic disturbances [1].

ii) During the period December 1988 - November 1989, the 2-D numerical algorithm was extended to the study of 3-D supersonic shear layers. The 2-D and 3-D computer codes were distributed to interested researchers (Dr. S. Ragab of VPI [Ref. 2] and Dr. Chris Tam of Florida State University). The 3-D code was used to study the response of a 3-D spatially periodic, temporally growing shear layer to random initial disturbances. The convective Mach number was parametrically varied. It was observed that the 3-D shear layer has characteristics very similar to the 2-D shear flows, in that the temporal growth of kinetic energy associated with the disturbances decreased as the convective Mach number was increased. No explicit turbulence model was used in this study.

During the current reporting period (December 1, 1989 - May 31, 1990), the above calculations have been repeated, but using initial disturbances based on linear stability theory, rather than random initial disturbances. The objective has been to investigate if suitable combinations of streamwise and spanwise disturbances will lead to temporal (or spatial growth) of shear layers that reaches an asymptotically constant value as the convective Mach number is increased. It may be recalled that Papamoschou and Roshko reported an asymptotic spatial growth rate of shear layers that is 0.3 times that of an incompressible shear layer as the convective Mach number is increased. Thus our investigation during this phase of the study has been to determine if 3-D instability waves are responsible for this growth rate. 2-D Linear stability analyses by other researchers as well as 2-D numerical simulations by the present researchers have shown that the spatial growth rate goes to zero as the convective Mach number increases beyond 1.5 or so.

THE WORK PERFORMED THROUGH THE REPORTING PERIOD

We have studied the temporal evolution of shear layers disturbed with 3-dimensional waves at two different convective Mach numbers. The mode shapes of the most amplified waves for a given Mach number and mean velocity profile have been obtained with a linear stability analysis code [Ref. 3]. We have employed the following hyperbolic tangent mean velocity profile throughout this study:

$$U = (U_1 + U_2)/2 + (U_1 - U_2)/2 \tanh(z)$$

where U_1 and U_2 are the velocities at the upper and lower bounds of the shear layers, respectively. The disturbance waves are given by the linear stability analysis for the velocity components, density and temperature in the following form:

$$d(x,y,z,t) = A\, D(z) \exp\{ i(\alpha x + \beta y - \omega t)\}$$

where D(z) is the eigenfunction, α and β are the wave numbers $Im(\omega)$ is the exponential growth rate. A is the magnitude of the disturbances superimposed onto the main flow. The eigenfunctions are normalized with respect to the maximum streamwise velocity disturbance.

The flowfield is initialized with the disturbance waves superimposed onto the mean velocity profile at time $t = 0$. Since we are interested in the temporally evolving shear layers, periodic boundary conditions are applied in the stream and spanwise directions while the slip-wall condition is imposed in the cross-stream direction.

The computational domain is a rectangular channel which extends over one wavelength of the longest disturbance wave in stream and spanwise directions, that is $x_{max} = 2\pi/\alpha$ and $y_{max} = 2\pi/\beta$. (x and y are the stream and spanwise directions, respectively.) In the cross-stream direction it extends from -7.5 to 7.5. The computational domain was discretized with a 66 x 34 x 121 uniformly spaced grid along the streamwise, spanwise and normal (wall to wall) directions, respectively. The computations were carried out on the CRAY-XMP at the Naval Research Laboratory while the post processing of the results was done locally. Computations for a typical case took approximately 45 minutes of CPU time.

In the first case, case A, U_1 and U_2 are set to 0.7 and -0.7 respectively, which sets the convective Mach number to 0.7. Two waves, one being a neutral, 2-dimensional one with $\alpha = 0.6$, $\beta = 0.$, $Im(\omega) = 0.0157$ and the second one being 3-dimensional, the most amplified wave which is incidentally the subharmonic of the first one with $\alpha = 0.3$, $\beta = 0.3$, $Im(\omega) = 0.084$ were superimposed onto the mean flow.

The amplitude of the primary wave is set to be 5% of the mean flow velocity, whereas that of the subharmonic one is set to 1.5 %. The development of the flow in time is shown in Figures 1-2 in terms of vorticity contours along the stream and spanwise directions. It should be noted that the growing strength of streamwise vorticity indicates the growth of the subharmonic wave while strength of the spanwise vorticity field stays approximately at the same level. Figure 3 shows the modal kinetic energy content of the flow field, which is defined as

$$E_{mn}(t) = [\mathbf{u}\mathbf{u}^* + \mathbf{v}\mathbf{v}^* + \mathbf{w}\mathbf{w}^*]\, dz$$

where **u**,**v** and **w** are the Fourier transforms of the velocity field (u,v,w) on the x-y plane and the symbol '*' denotes the complex conjugate, and the integration is from

the lower wall to the upper wall. It is observed in Fig. 3 that the kinetic energy of the modes (2,1) and (1,1) grows in time substantially while the neutral mode (2,0) stays almost constant.

Figure 4 compares the computed growth of the subharmonic mode, mode (2,1), to the one predicted by the linear stability theory, namely ω. The computed growth rate initially agrees well with the linear theory but rapidly departs from linear theory at later time levels.

In the second case, Case B, we studied the instabilities in supersonic shear layers, where $U_1 = 1.2$ and $U_2 = -1.2$. A single 3-dimensional wave with $\alpha = 0.14$, $\beta = 0.07$ and $Im(\omega) = 0.018$ is superimposed onto shear layers. The magnitude of these disturbances is set to $0.15\ U_1$. The development of the computed flowfield is shown in Figs. 5-6 in terms of stream and spanwise vorticity contours. The change in the pressure field at the mid-shear plane is also given in Fig. 7. In these figures it is observed that the initial 3-dimensional disturbance field tends to orient itself in the streamwise direction. This behavior is also supported by the change in the modal energy content of the flowfield given in Fig. 8. The initial growth in the (1,1) energy mode which corresponds to the mode of the introduced disturbance, is followed by its decrease while the two-dimensional modes (1,0) and (2,0) get energized.

Figure 9 shows the growth of the (1,1) modal energy for the convective Mach number 1.2 case. For comparsion, the growth of modal energy for the lower convective Mach number case ($M=0.7$) is plotted on the same graph. It is clear that as the convective Mach number increases, the growth of the energy contained in the perturbations decreases.

CONCLUDING REMARKS

To date, the limited number of calculations we have done for 3-D supersonic shear layers as well as linear stability analyses by other researchers indicates that the growth rate (temporal or spatial) of supersonic shear layers decreases to zero as the convective Mach number increases beyond 1.5 or so. This is in contrast to Papamoschou and Roshko's observations. Two possible reasons have been sited for the discrepancy between linear stability analyses or laminar simulations and experiments. Some researchers (e.g. Chris Tam of Florida Stat U.) feel that the

transverse modes associated with the reflection of acoustic waves off the top and bottom wind tunnel walls may play a role in the sustained growth of shear layer in Papamoschou's experiments. There is also some indication that turbulence plays a role in promoting mixing and sustaining the shear layer growth [Ref. 4]. During the remaining portion of the project, the present investigators plan to study the growth of 2-D and 3-D supersonic shear layers using a Reynolds stress turbulence model described in Ref. 4 and successfully used to simulate Papamoschou's experiments. This model has already been coded by Mr. Ray Hixon, a graduate student, and incorporated into our 2-D shear flow code. The goal of this investigation is to determine if and how compressibility influences the Reynolds stress production and turbulence dissipation rate in 2-D and 3-D supersonic shear flows.

REFERENCES

1. Tang, W., Sankar, N.L. and Komerath, N., "Numerical Simulations of the Growth of Instabilities in Supersonic Free Shear Layers", AIAA paper 89-0376, To be published in Journal of Propulsion and Power, 1990.

2. Ragab, S. A. and Wu J. L., "instabilities of Supersonic Shear Flows", AIAA paper 90-0712.

3. Ragab, S., "linear Instability waves in supersonic Turbulent Mixing layers", AIAA paper 87-1418.

4. Sarkar S. and Balakrishnan L., " Application of a Reynolds Stress Turbulence model to the Compressible Shear Layer", ICASE report 90-18, Feb. 1990.

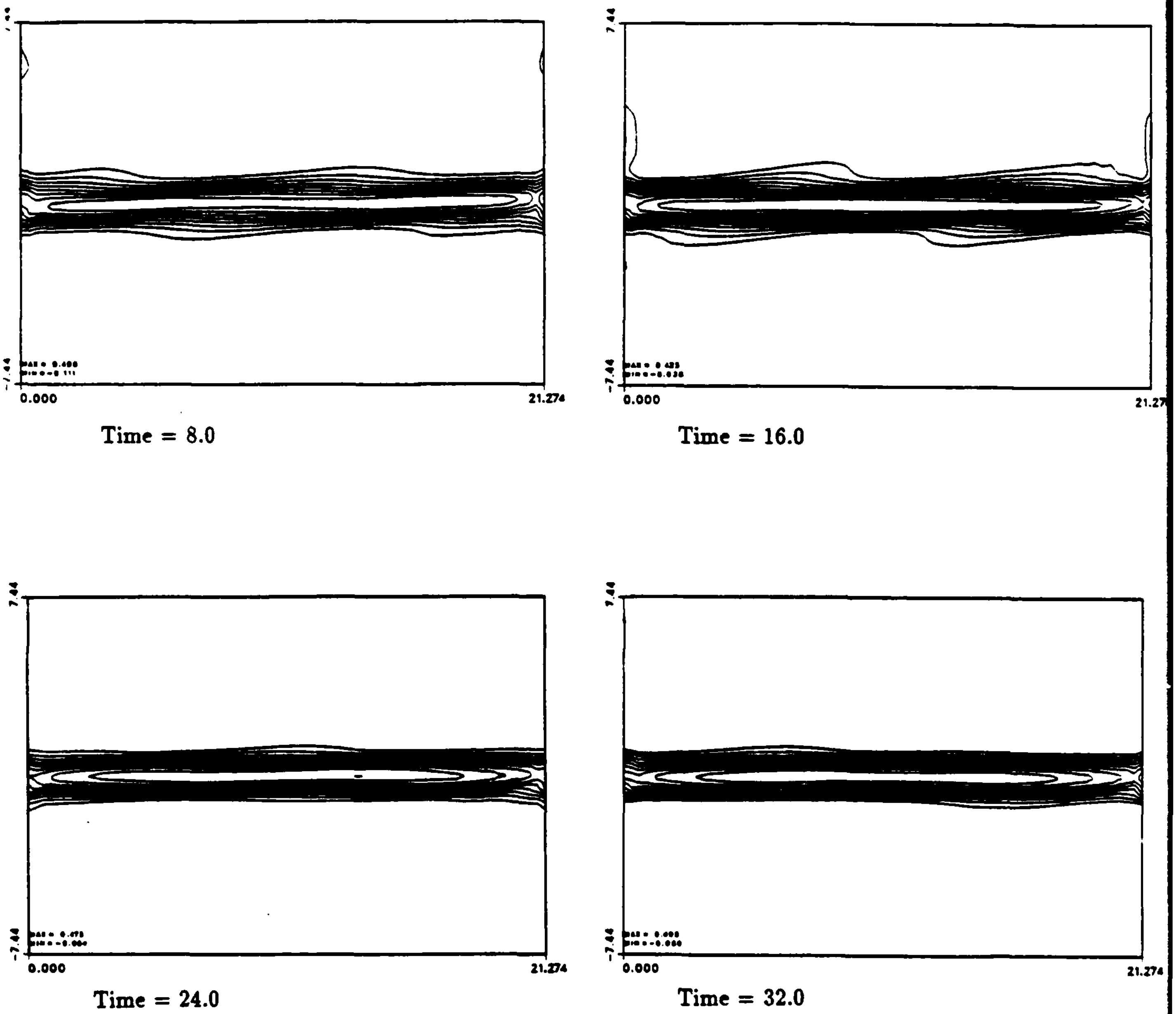


Figure 1. Spanwise vorticity contours at mid-span section - Re = 200, M = 0.7

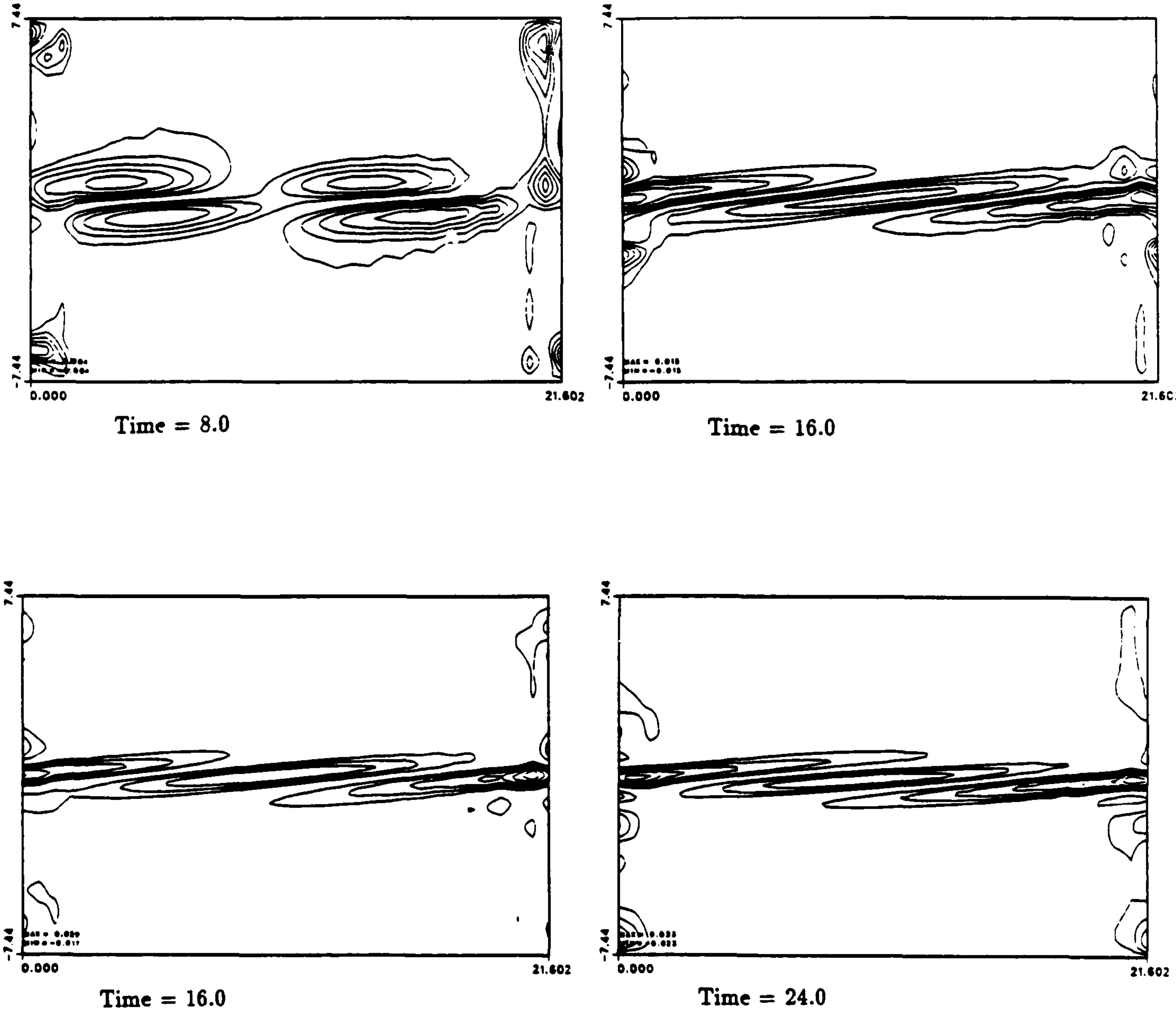


Figure 2. Streamwise vorticity contours at middle section - Re = 200, M = 0.7

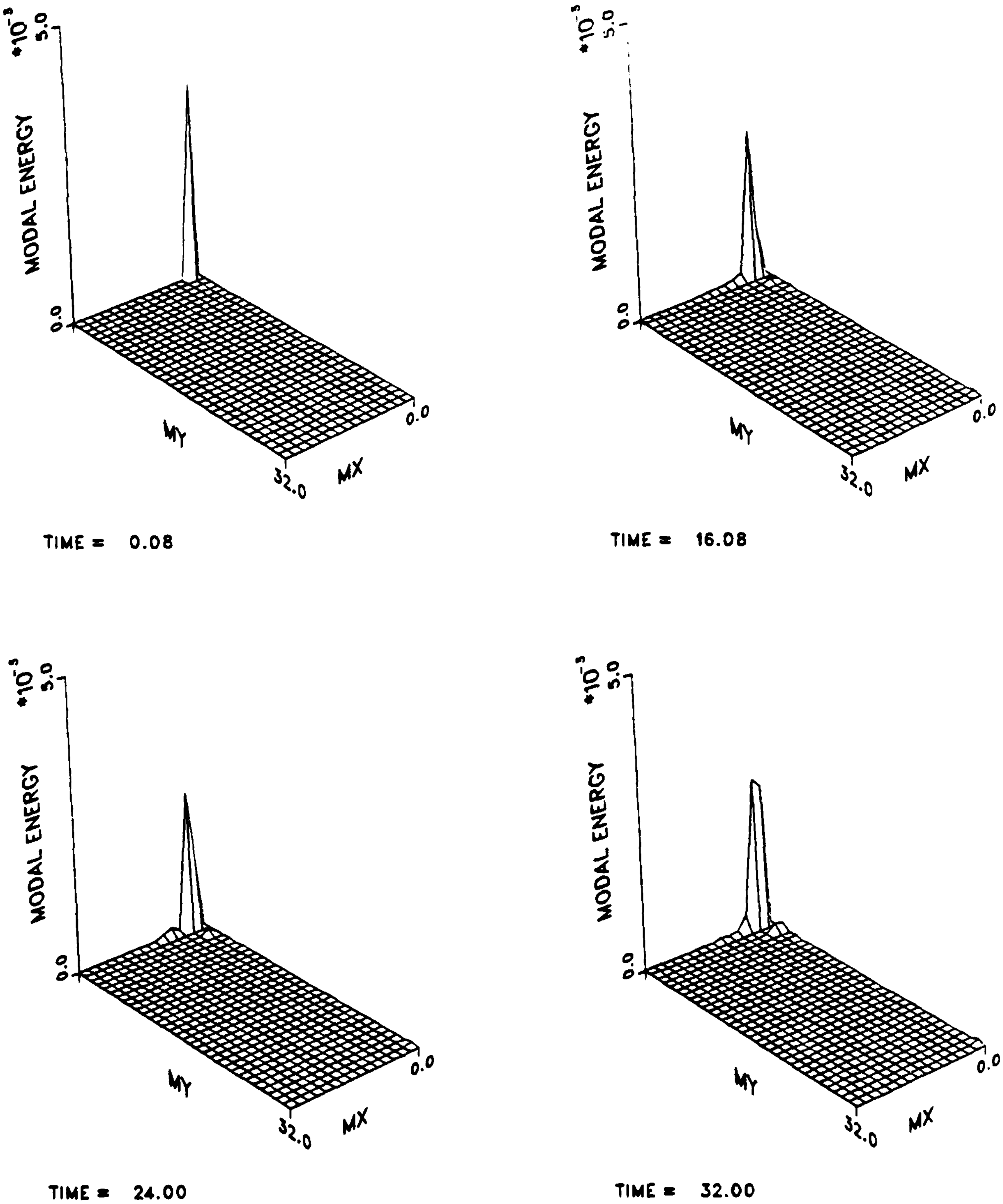


Figure 3. Modal energy content of the flow field - Re = 200, M = 0.7

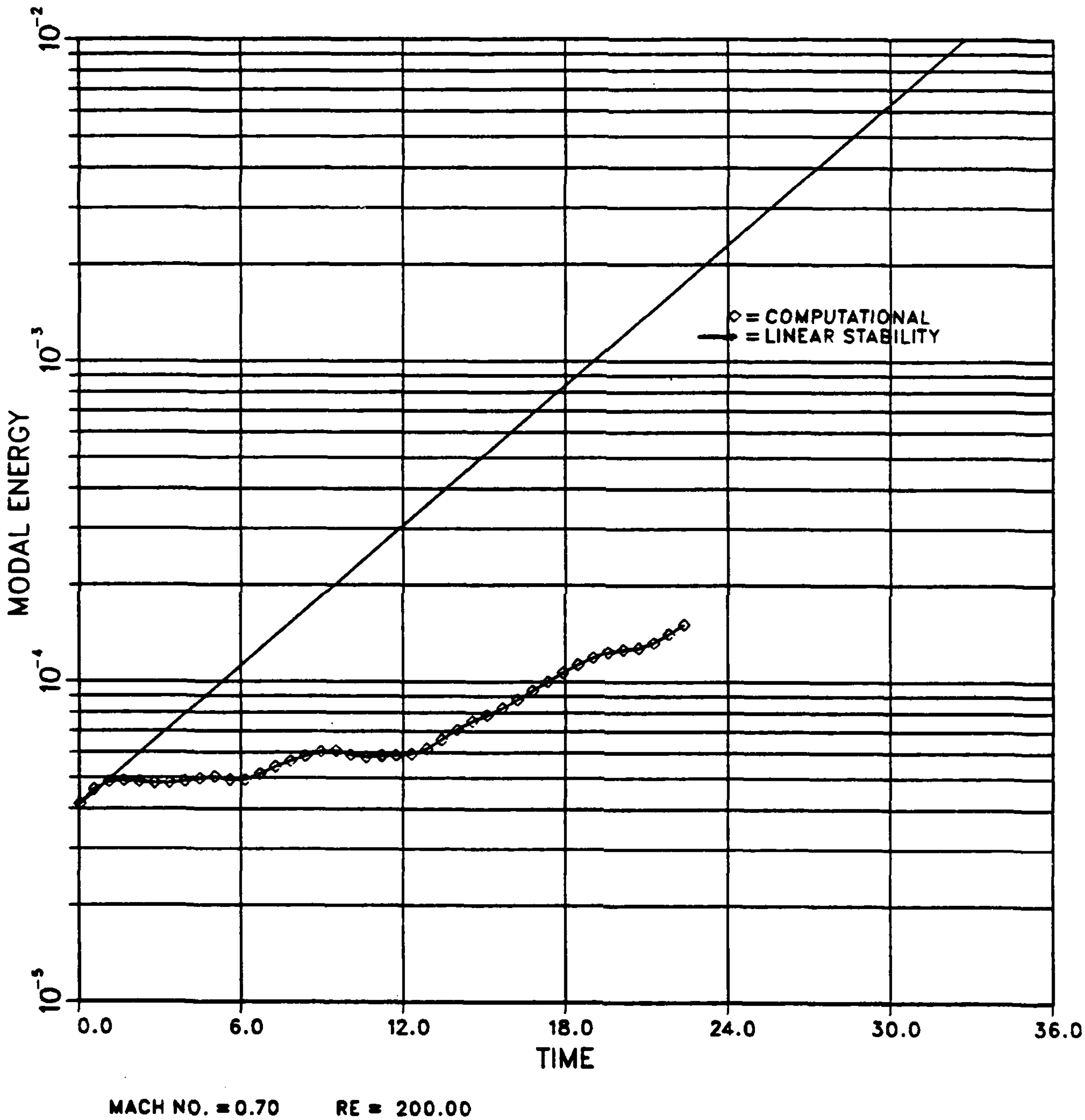


Figure 4. Time history of the (2,1) energy mode - Re = 200, M = 0.7

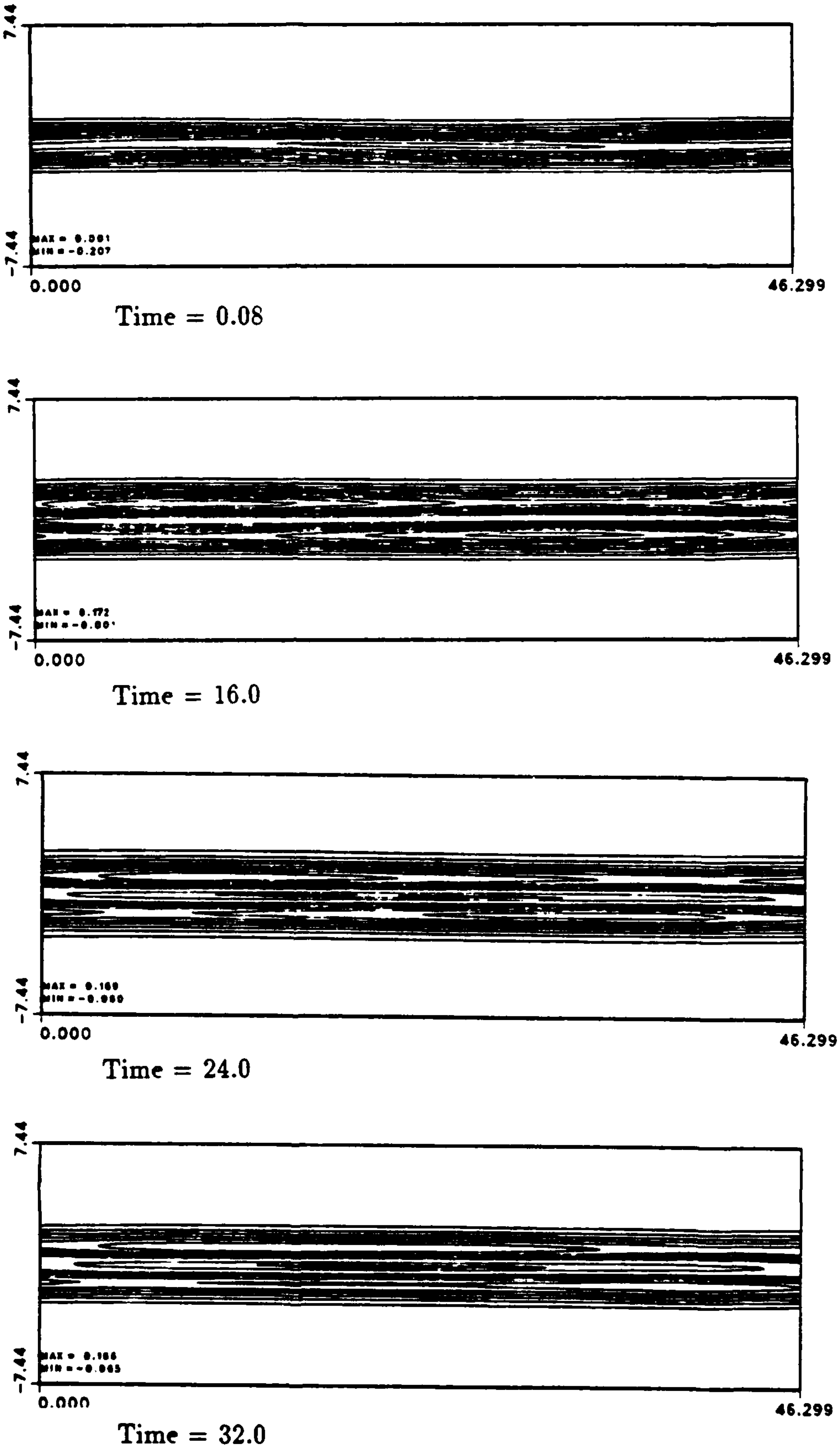
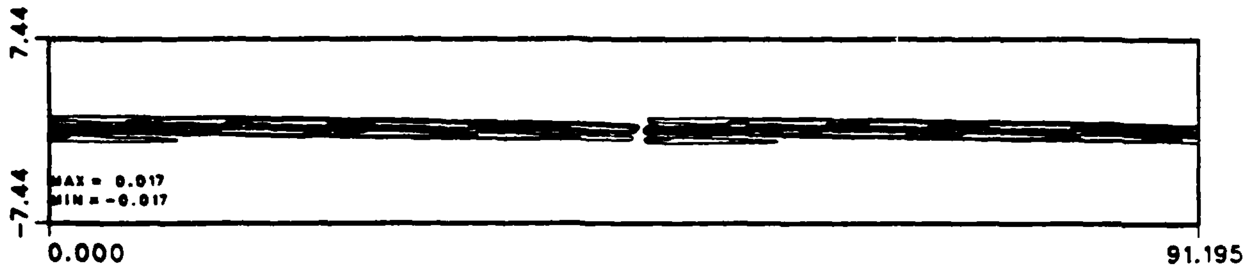


Figure 5. Spanwise vorticity contours at mid-span section - Re = 600, M = 1.2



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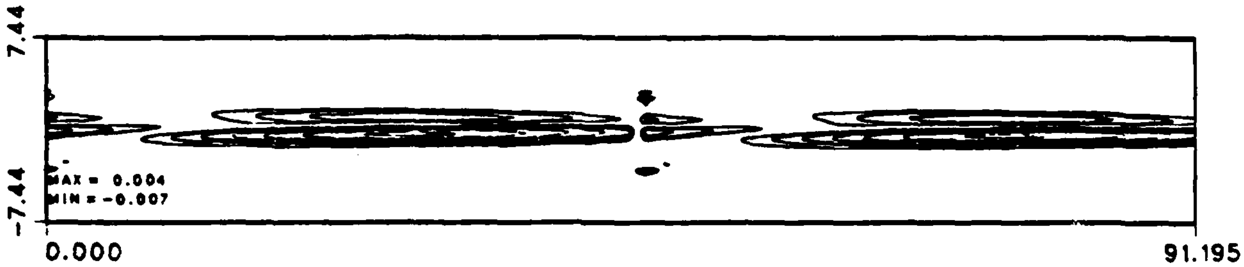

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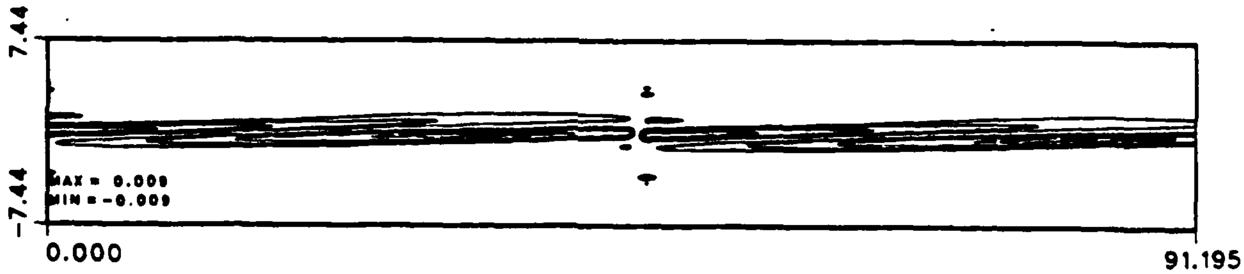

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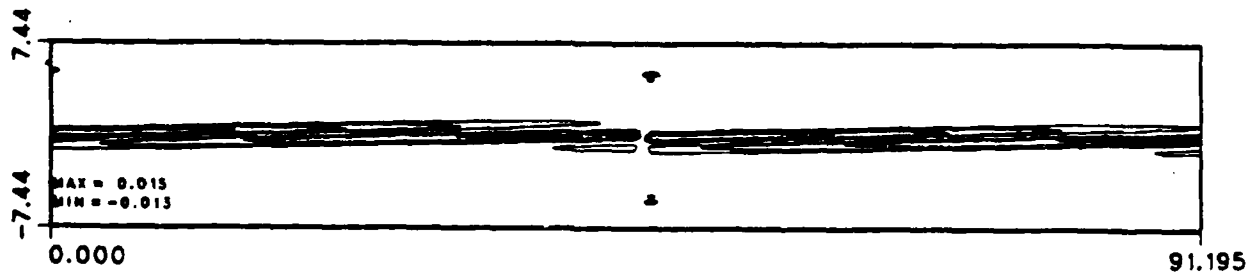

Time = 32.0

Figure 6. Streamwise vorticity contours at middle section - Re = 600, M = 1.2

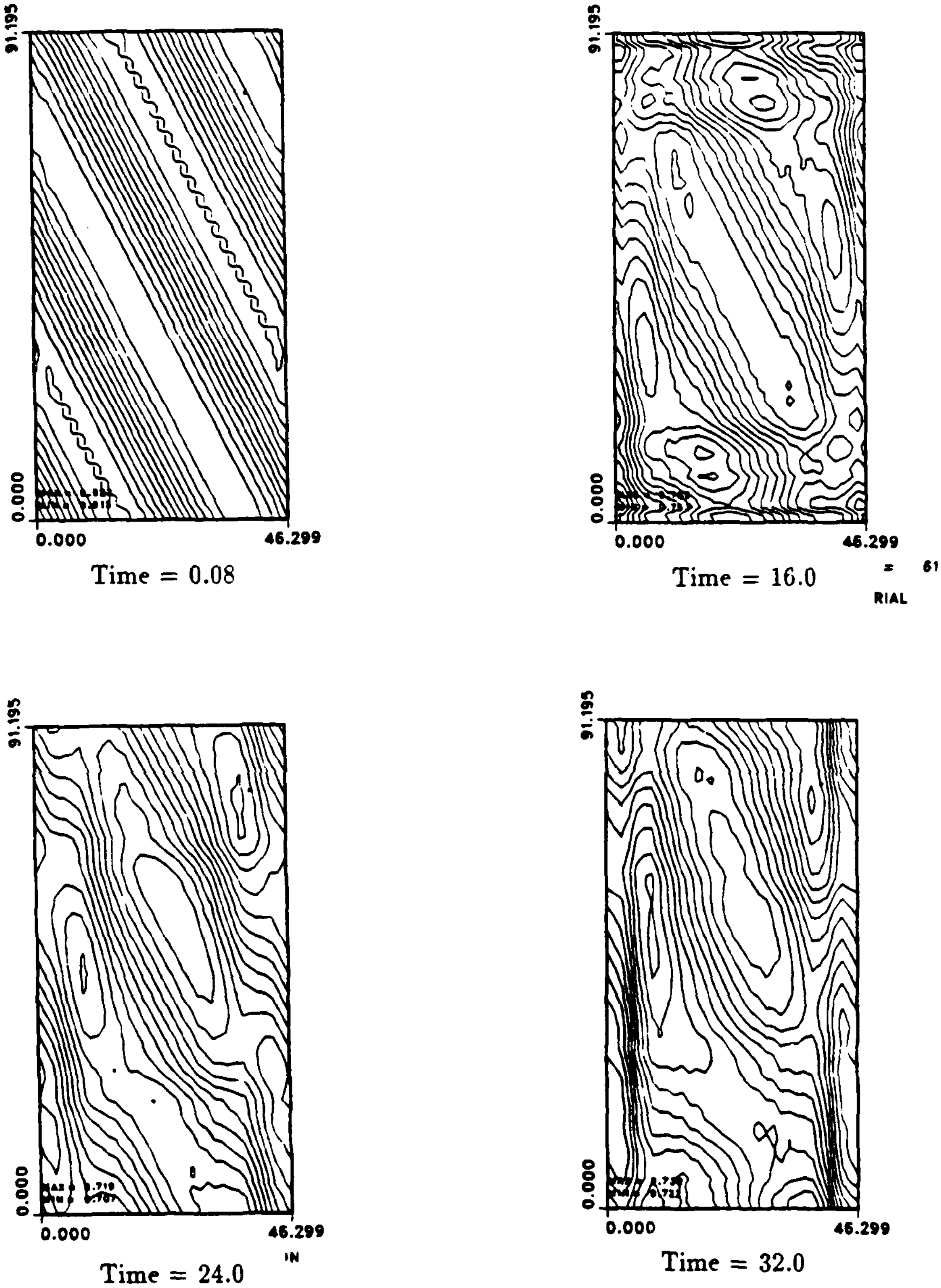

Figure 7. Pressure contours at the mid-shear plane - Re = 600, M = 1.2

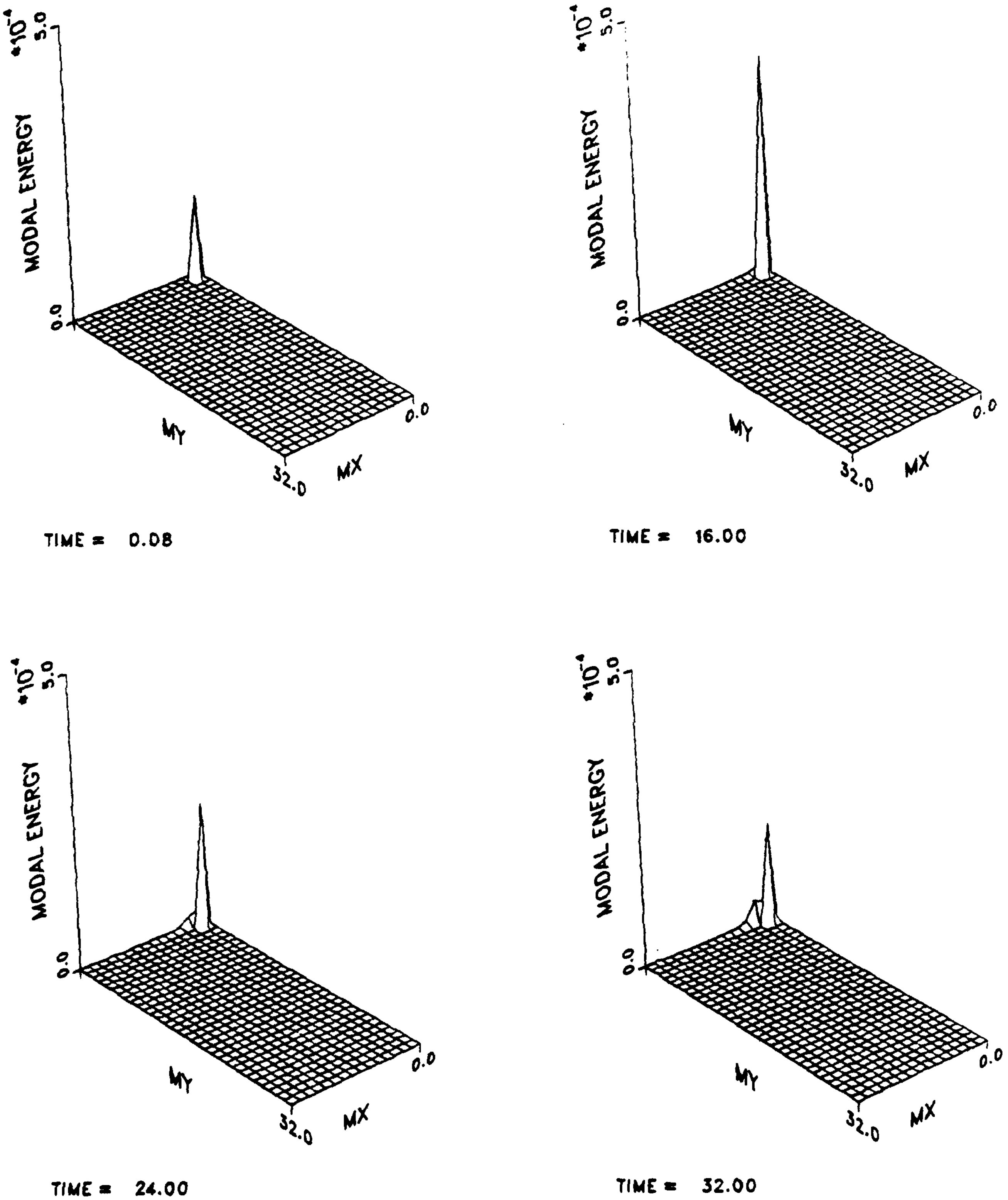

Figure 8. Modal energy content of the flow field - Re = 600, M = 1.2

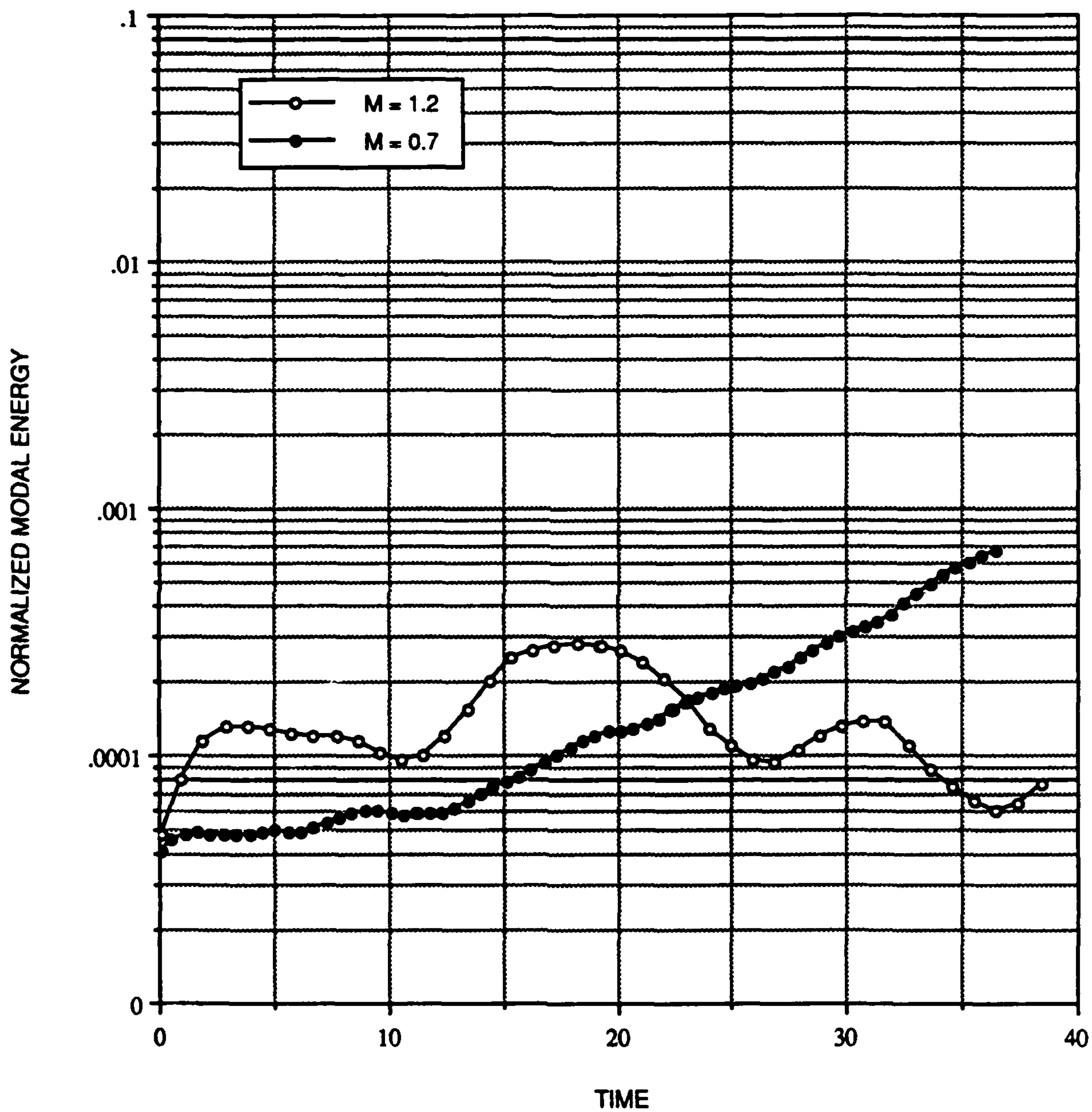

Figure 9. Time history of the (1,1) energy mode - Re = 600, M = 1.2